A naturalistic justification of the Generic Multiverse with a core

Matteo de Ceglie

decegliematteo@gmail.com

Paris Lodron Universität Salzburg

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Structure of the Presentation

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Introduction outline

- Pluralism, Anti-Pluralism and Naturalism
- The reasons of the emergence of the multiverse:
  - Independent propositions;
  - Alternative set theories.
- A brief sketch of the main argument

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Pluralism, Anti-Pluralism and Naturalism

Naturalism

Mathematical practice should be considered the final judge for questions in philosophy of mathematics.

Anti-Pluralism

There is only *one* set theoretic universe.

Pluralism

There are various set theoretic universes.

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- The multiverse is just as good, when dealing with actual mathematical practice, as the single universe;
- Moreover, in the multiverse is possible to prove more things than in the single universe;
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- The Multiverse conceptions in set theory
  - The broad multiverse;
  - The Generic Multiverse with a core (GM_H)
- The naturalistic approach

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Multiverse conceptions in set theory

The *broad* multiverse

All the possible universes are part of the multiverse, with no hierarchy nor criterion to sort them.

The generic multiverses

In this kind of multiverses we differentiate between universes using a strong logic (an idea owed to Woodin, from now on GM_Ω) or supposing the existence of a core (an idea owed to Steel, that is the GM_H).

The hyperverse

A multiverse of multiverses.

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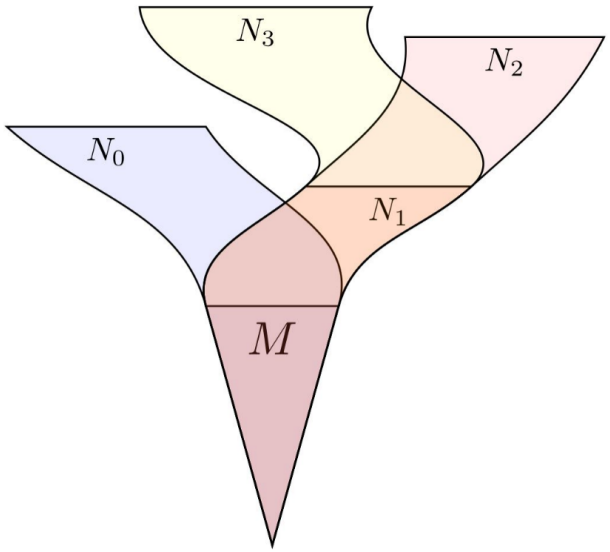
The Generic Multiverse with a core (GM_H)

Definition of the *core*

The core of the multiverse is the collection of all the statements that are true in *every* universe of the multiverse. We can then consider every universe of the generic multiverse and *extension* of its core.



The naturalistic approach

The problem

Which is best for mathematical practice? The Single Universe framework or the Multiverse?.

UNIFY

Our framework should be *foundational*.

MAXIMIZE

The framework theory should be as powerful as possible, not restricting in any way the development of the foundations of mathematics (the framework theory should be the most Generous Arena for mathematics).

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Some more about the naturalistic approach

- We could further refine UNIFY defining the following *foundationality* feature:
  - Meta-mathematical Corral;
  - Elucidation;
  - Shared Standard;
  - Risk Assessment.
- A candidate framework for mathematical practice should at least provide all these features.

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- The foundationality of the GM_H
- Maximizing the descriptive power of the GM_H
- The non-foundationality of the other multiverse conceptions

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The foundationality of the GM_H

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- Given this, there would be no reason to switch from the Single Universe to the GM_H.

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Maximizing the power of the GM_H

- Lets suppose that our multiverse is composed by only two universes: one is a model of ZFC and the other a model of $ZF + AD$;
- In the multiverse, we retain all the results and true statements of ZFC *and* all the results of $ZF + AD$;
- Moreover, we can also prove several more interesting isomorphisms in this simplified multiverse;
- On the other hand, in the Single Universe, we limit ourselves to only a subset of all the results we can prove in the multiverse;
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Against the other multiverse conceptions

- All the other multiverse conceptions are equally powerful from the MAXIMIZE point of view;
- Although, they all fail the foundationality test:
  - The broad multiverse fails to provide Shared Standard and Risk Assessment;
  - Mochizuki's $\mathcal{IUTT}$ fails to provide us Meta-mathematical Consilience;
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